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NOTE ON THE ERRORS APPEARING IN THE BOOK "CONFLUENT HYPERGEOMETRIC FUNCTIONS" BY L. J. SLATER

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Erroneous formulas of expansion of the Kummer and Whittaker functions of the first kind in terms of cylindrical functions given in a monograph [1] by Slater, are corrected.

Expansions of functions ${}_1F_1\,(a,\ b,\ x)$ and $M_{k,m}\,(x)$ into series in cylindrical functions are found useful when tables covering the required interval of variation of parameters are not available. Slater gives such expansions in [1], unfortunately all four formulas appearing in their final form in Sect. 2.7.3 are erroneous.

Fallacy of Formulas (2.7.14) and (2.7.16) becomes obvious on applying them to already known cases. Indeed, when $a = n + 1/2$ and $b = 2n + 1$, Formula (2.7.14) yields

$${}_1F_1\,[n + 1/2,\ 2n + 1,\ x] = 2^{2n}\,\Gamma(n)\,e^{1/2x}x^{-n}I_n\,(1/2x)$$

which contradicts the particular Formula (2.7.1). On putting $k = 0$, Formula (2.7.16) yields

$$M_{0,m}\,(x) = 2^{2m}\Gamma(m)x^{1/2}I_m\,(1/2\ x)$$

which in turn contradicts the exact Formula (1.8.11) (see also (9.235) of [2]).

On checking we have found that the error was caused by the incorrect computation in [1] of the function ${}_0F_1\,(;b,\ x)$ on passing from Formula (2.7.10) to (2.7.14) and (2.7.15), and the function was transferred, as it stood, into (2.7.16) and (2.7.17). Since the same error appears in all four formulas, we shall compute ${}_1F_1\,(a,\ b,\ x)$ and give the exact result for $M_{k,m}\,(x)$.

By (2.7.11) we find

$${}_0F_1\,[;\, b - a + 1/2 + n,\ (1/4x)^2] = \Gamma(b - a + 1/2 + n)\,(1/4x)^{a-b+1/2-n}I_{b-a-1/2+n}\,(1/2x)$$

Using the relation $\Gamma(b-a+{}^1/_2+n)=\Gamma(b-a-{}^1/_2)(b-a-{}^1/_2)_n(b-a-{}^1/_2+n)$

we shall insert in (2.7.10) the expression obtained for ${}_0F_1$. Simplifying we obtain the following exact expansion
$$ {}_1F_1[a,b,x]=e^{{}^1/_2x}\Gamma(b-a-{}^1/_2)({}^1/_4x)^{a-b+{}^1/_2}\times $$
$$ \times\sum_{n=0}^{\infty}\frac{(2b-2a-1)_n(b-2a)_n(-1)^n}{n!\,(b)_n}(b-a-{}^1/_2+n)\,I_{b-a-{}^1/_2+n}({}^1/_2x) $$
from which we see that the factor $(b-a-{}^1/_2+n)$ is missing from the expressions under the summation sign in (2.7.14) and (2.7.15).

Writing now the Whittaker function $M_{k,m}(x)$ according to Formula (1.6.4) we obtain the following correct form of the expansion:
$$ M_{k,m}(x)=2^{2m+2k}x^{{}^1/_2-k}\Gamma(m+k)\times $$
$$ \times\sum_{n=0}^{\infty}\frac{(2m+2k)_n(2k)_n(-1)^n}{n!\,(1+2m)_n}(m+k+n)\,I_{m+k+n}({}^1/_2x) $$
Comparing it with (2.7.16) and (2.7.17) in [1] we see that the factor $(m+k+n)$ is missing from the latter.

We can easily see that our expansions yield, correctly, the particular cases (2.7.1) and (1.8.11) of [1].

We also note a few misprints appearing in [1]. In the right hand sides of (1.5.4) and (1.5.5) the factor x^c should read x^{c-1}, although it does not affect the argument that follows. Last formula of Sect. 2.7.2 on page 32 has the factor $(2n+1)_m$ appearing in the denominator under the summation sign, wnich should read $(2n+t)_m$.

All errors and misprints discussed above appear also in the English original [3].

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